**NTU SSS Economics HE2001**  
**Tutorial 9 (Perfect Competition and Welfare)**

1) Consider an Edgeworth box with 2 goods, apples and oranges. There are two consumers Alice and Bob. Alice has an endowment of 10 oranges while Bob has an endowment of 10 apples. Both Alice and Bob have Cobb-Douglas Utility .

a) For prices and , derive the demands of Alice and Bob.

b) Given the above demands which are a function of prices, write out the condition for market clearing.

c) Show that the competitive equilibrium allocation in this exchange economy is where Alice and Bob have 5 oranges and 5 apples each. Illustrate the competitive equilibrium in an Edgeworth Box, showing how it is pareto efficient.

d) Show that the allocation of 9 oranges and 9 apples to Alice and 1 orange and 1 apple to Bob is also pareto efficient. (Hint: try drawing the indifference curves through the point).

e) Give two examples of a redistribution of the original endowment which will result in the same pareto efficient outcome in a competitive equilibrium.

2) Linus Straight’s utility function is , where a is his consumption of apples and b is his consumption of bananas. Lucy Kink’s utility function is . Lucy initially has 12 apples and no bananas. Linus initially has 12 bananas and no apples.

1. Draw an Edgeworth box, showing the initial allocation and sketching in a few indifference curves. Measure Lucy’s consumption from the upper right corner and Linus’s from the lower left corner. Draw a line through all of the Pareto optimal allocations.
2. In this economy, in competitive equilibrium, what will be the ratio of the price of apples to the price of bananas ?

1. Find the quantities of apples and bananas consumed in competitive equilibrium by Linus and Lucy.

3) Paul and David consume apples and oranges. Paul’s utility function is and David’s utility function is , where and are apple consumptions for Paul and David, and and are orange consumptions for Paul and David.

There are a total of 12 apples and 12 oranges to divide between Paul and David.

(a) Draw an Edgeworth box showing some of their indifference curves. Mark the Pareto optimal allocations on your graph.

(b) Derive the set of envy-free allocations using the following procedure.

* 1. Write one inequality that says that Paul likes his own bundle as well as much as he likes David’s.
  2. Write another inequality that says that David likes his own bundle as much as he likes Paul’s.
  3. Use the fact that at feasible allocations, 𝐴𝑃 +𝐴𝐷 = 12and 𝑂𝑃 +𝑂𝐷 =12to rewrite the above inequalities in terms of 𝐴𝑃 and 𝑂𝑃 for Paul and 𝐴𝐷 and 𝑂𝐷 for David.
  4. Indicate the set of allocations which satisfy **both** of these inequalities in the figure.

(c) In the Edgeworth box, denote the set of fair allocations.

4) Consider a small exchange economy with two consumers, Astrid and Birger, and two commodities, herring and cheese. Astrid’s initial endowment is 4 units of herring and 1 unit of cheese. Birger’s initial endowment has no herring and 7 units of cheese. Astrid’s utility function is . Birger is a more inflexible person. His utility function is .

(Here and are the amounts of herring and cheese for Astrid, and and are amounts of herring and cheese for Birger.)

(a) Draw an Edgeworth box, showing the initial allocation and sketching in a few indifference curves. Measure Astrid’s consumption from the lower left and Birger’s from the upper right. In your Edgeworth box, draw two different indifference curves for each person.

(b) Illustrate on the same diagram, the locus of Pareto optimal allocations.

(c) Let cheese be the numeraire (with price 1) and let denote the price of herring. Derive the allocations and prices in the competitive equilibrium. (Hint follow the steps in the lecture and question 1)

**Sample Question (No solutions will be provided for these)**

Consider a small exchange economy, with two consumers, Iris and Joseph, and two goods and . Iris is endowed with 15 units of and 3 units of . Joseph is endowed with 5 units of and 7 units of .

Iris has utility while Joseph has utility .

1. Draw the Edgeworth box for this exchange economy as accurately as possible, indicating the endowment and drawing several indifference curves for Iris and Joseph. **(7 marks)**
2. In the figure you have drawn in (a), draw the indifference curves through the endowment and indicate where a competitive equilibrium must lie. Explain. **(7 marks)**
3. Is the allocation where Iris gets while Joseph gets a fair allocation? Explain.  **(8 marks)**
4. “There always exists an initial endowment for Iris and Joseph where the allocation in the consequent competitive equilibrium is fair for any (standard) preferences of Ivy and Joseph.” True or False? Explain. **(8 marks)**